Fatigue Reliability of Gas Turbine Engine Components under Scheduled Inspection Maintenance

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A probabilistic method is developed for the fatigue reliability analysis of gas turbine engine components under scheduled inspection maintenance in service. Various statistical uncertainties involved in the complex design system of gas turbine engine components have been taken into account, including the time to crack initiation, fatigue crack propagation, service loads, crack modeling, crack geometry, nondestructive evaluation (NDE), etc. It is demonstrated that the service inspection maintenance can be used to improve the reliability of fatigue-critical components significantly. Such an improvement in fatigue reliability is shown to depend on the capability of the NDE system employed. An example for the third-stage turbine disk of a TF-33 jet engine has been worked out to demonstrate the application of the analysis methodology developed.

I. Introduction

MOST rotor disks of gas turbine engine are limited by low-cycle fatigue (LCF) life, generally expressed in terms of mission equivalency cycles. The LCF life usually is defined as the number of cycles necessary to initiate a crack approximately 0.03 in. (0.76 mm) long. The distribution of LCF life is obtained for a given set of loading conditions (stress/strain, time, temperature). Traditionally, the LCF life prediction for gas turbine rotor disks is made for crack initiation life at an occurrence rate of 1 in 1000 disks. It is at this life that all LCF-limited disks are removed from service. This procedure has been very successful in preventing the occurrence of castastrophic failure of disks in the field. However, in retiring 1000 disks because 1 may fail, the remaining life of the 999 unfailed disks is not utilized.

Recently, guidelines and requirements for the structural design of gas turbine engine components are given in Engine Structural Integrity Program, referred to as ENSIP.[1-3] Furthermore, the philosophy of retirement for cause (RFC) has been proposed,[4-8] under which each disk is inspected periodically. When a crack is detected during inspection, the disk is retired; otherwise, it is returned to service until the next inspection maintenance. This procedure could be repeated until the disk has incurred detectable damage (crack), at which time it is retired for that reason (cause). Retirement for cause (RFC) is, then, a methodology under which an engine disk is retired from service when it has incurred quantifiable damage, rather than because an analytically determined minimum design life had been exceeded.

To implement the RFC procedures, an optimal inspection interval should be determined such that the life-cycle cost of engine components is minimum and a high level of reliability is maintained. While the fatigue reliability of airframe structures under scheduled inspection and repair maintenance or periodic proof test maintenance has been investigated,[9-17] the reliability analysis of gas turbine engine components has received considerable attention recently. An analysis methodology is developed in this paper to determine the fatigue reliability of gas turbine engine disks under scheduled inspection maintenance in service as shown in Fig. 1. In addition, the average percentage of components to be replaced during each service inspection maintenance is estimated. The present methodology is based on the probabilistic approach taking into account all the statistical variables in the complex RFC system, including the time to crack initiation, fatigue crack growth damage accumulation, service loads, temperature profiles, fabrication deviations, nondestructive evaluations, etc. An example using the third-stage turbine disk of a TF-33 jet engine is given to demonstrate the application of the analysis methodology. The current investigation represents one phase of the probabilistic retirement-for-cause analysis.[18-19]

II. Preliminary

Initial Fatigue Quality

One important quantity in the fatigue analysis is the initial fatigue quality (IFQ), which defines the initially manufacturing state prior to service of a component or details, such as bolt holes, rim holes, cooling air holes, web holes, etc. For some engine materials, such as titanium, Waspaloy, Astroloy, etc., the initial fatigue quality can be represented by the statistical distribution of the time to crack initiation. The time (cycles) required to initiate a reference crack size a_0 (e.g., 0.03 in.), referred to as time (cycles) to crack initiation (TTCI), is also referred to as the low-cycle fatigue (LCF) life in gas turbine engine literature; it involves considerable statistical variability.

The time to crack initiation, denoted by T, at a critical location of a component under service loading spectra is assumed to follow the lognormal distribution

$$f_T(t)=\frac{\log e}{\sqrt{2\pi}t\tilde{\sigma}_T}\exp\left[-\tfrac{1}{2}\left(\frac{\log t-\tilde{\mu}_T}{\tilde{\sigma}_T}\right)^2\right] \tag{1}$$

where $f_T(t)$ is the probability density function. The mean value $\tilde{\mu}_T$ of log T is obtained from a few component test data under nominal load spectra. The standard deviation $\tilde{\sigma}_T$ of log T is computed from the constant-amplitude tests results of coupon specimens, as well as the statistical dispersion of service loads.[18]

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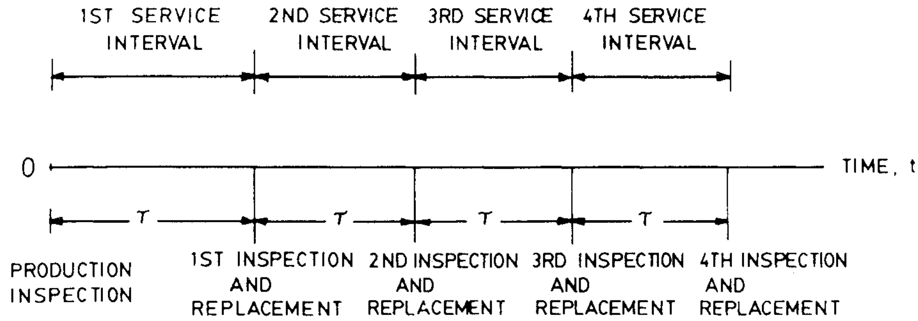

Fig. 1 Scheduled inspection maintenance.

Fatigue Crack Propagation

After a crack is initiated at the reference crack size a_0, it will propagate under service loads. The Paris model for the crack growth rate of a corner crack has been modified as follows[18]:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = QS^V a^b \tag{2}$$

where $\mathrm{d}a/\mathrm{d}t$ = crack growth rate; Q, V, and b are constants; and S is the "normalized" maximum stress in the design spectrum, in the sense that $S = 1$ represents the nominal design spectrum.

The values of Q, V, and b can be determined from three or more crack growth curves, i.e., crack size $a(t)$ vs cycles t, using the least-squares best-fit procedures. The crack growth curves can be obtained from the results of spin pit tests, field experience, or numerical integrations using general computer programs under spectrum loading. It was demonstrated in Refs. 18 and 19 that the crack growth rate model proposed in Eq. (2) correlated very well with available crack growth damage results in bolt holes and cooling air holes of engine disks.

Under well-controlled laboratory environments, test results of the crack growth rate for engine materials (such as IN100, Waspaloy, etc.), exhibit considerable statistical variability.[20,21] Likewise, many quantities influencing the crack growth behavior also involve statistical scatter. The statistical variability of the crack growth damage accumulation should be taken into account. Recently, efforts have been made to randomize the deterministic crack growth rate equation.[20-22] The log normal crack growth rate model is employed herein to randomize Eq. (2) as[20-22]

$$\frac{\mathrm{d}a}{\mathrm{d}t} = ZQa^b \tag{3}$$

where Z is a random variable accounting for the following contributions to the scatter of the crack growth rate,

$$Z = H_1 H_2 H_3 H_4 S^V \tag{4}$$

where H_1, H_2, H_3, H_4, and S are random variables denoting the contributions to the statistical variability of the crack growth rate from various sources. H_1 represents the material variability; H_2 the variability of crack geometry, such as aspect ratio, stress intensity factor, etc.; H_3 the variability of crack modeling, e.g., two-dimensional crack model, etc.; and H_4 the variability of crack growth damage due to each equivalent cycle. For instance, two stress records may be modeled by the same number of equivalent cycles, but the corresponding crack growth damages are not identical. Finally, S denotes the variability of stress resulting from variabilities of service loads, temperature profile, and the stress concentration factor K_t in each hole.

All of the random variables H_1, H_2, H_3, H_4, and S are assumed to follow the log normal distribution with the median value equal to unity (i.e., taking values around unity). It follows from Eq. (4) that the random variable Z is log normal with the median value $\tilde{Z}$ equal to unity, i.e., $\tilde{Z} = 1.0$. Thus, the deterministic crack growth rate equation (2) represents the median crack growth behavior, whereas the statistical variability of the crack growth rate is taken care of by the log normal random variable Z.

The mean value $\tilde{\mu}_z$ and standard deviation $\tilde{\sigma}_z$ of log Z is obtained from Eq. (4) as

$$\tilde{\mu}_z = \log \tilde{Z} = 0$$

$$\tilde{\sigma}_z = (\tilde{\sigma}^2_{H_1} + \tilde{\sigma}^2_{H_2} + \tilde{\sigma}^2_{H_3} + \tilde{\sigma}^2_{H_4} + V^2 \tilde{\sigma}^2_S)^{1/2} \tag{5}$$

where $\tilde{\sigma}_{H_1}$, $\tilde{\sigma}_{H_2}$, $\tilde{\sigma}_{H_3}$, and $\tilde{\sigma}_{H_4}$ are the standard deviations of log H_1, log H_2, log H_3, and log H_4, respectively, and $\tilde{\sigma}_s$ the standard deviation of log S.

Thus, the probability density function of Z, denoted by $f_Z(z)$, is given by

$$f_Z(z) = \frac{\log e}{\sqrt{2\pi} z \tilde{\sigma}_z} \exp\left[-\frac{1}{2}\left(\frac{\log z}{\tilde{\sigma}_z}\right)^2\right]; \quad 0 \le z < \infty \tag{6}$$

where $\tilde{\sigma}_z$ is given by Eq. (5).

Inspection Procedures

The capability of a nondestructive evaluation (NDE) system is defined by the probability of detection (POD) for all cracks of a given length in a given environment. Many POD curves can be found, for instance, in Refs. 23-27 and the references therein.

An engine component is inspected by two NDE systems using the intersection rule,[26,27] described as follows. When a component is inspected and accepted by the first NDE system, it is returned to service; otherwise, it is further inspected by the second NDE system. Some actions may be taken for those components rejected by the first NDE system before being inspected by the second NDE system—for instance, cleaning, polishing, or replicating of the rejected critical locations. The component is replaced only if it is rejected by both NDE systems; otherwise, it is returned to service. The second NDE system may be identical to the first one (i.e., the first NDE system is used twice) or it may be a redundant system to be used only for those components rejected by the first NDE system. However, the detection capability of the second NDE system should be much better than that of the first in order to return to service a high percentage of good components that are rejected by the first system. The surface preparation of the critical location, such as cleaning, polishing, or replicating, etc., will increase the NDE capability significantly.

The POD curve for the first NDE system is expressed by

$$POD(a;1) = \exp(\alpha^* + \beta^* \ln a)/[1 + \exp(\alpha^* + \beta^* \ln a)]; \quad 0 < a < \infty \tag{7}$$

where POD(a;1) is the probability of detecting the crack size a and α^* and β^* are constants. Equation (7) is referred to as the log odd function.[25,26]

The POD curve for the second NDE system is expressed in terms of the Weibull function, i.e.,

$$POD(a;2) = 0 \quad ; a < \epsilon$$
$$= 1 - \exp\{-[(a-\epsilon)/\beta]^\alpha\} \quad ; a \geq \epsilon \tag{8}$$

where ϵ is the crack size below which the crack cannot be detected by the NDE system and α and β are constants representing the bandwidth and central location of the POD curve, respectively. It should be mentioned that the POD curve for the second NDE system, POD (a;2), reduces to the unit step function, i.e., POD(a;2) = 0 for $a < a_{NDE}$ and POD(a;2) = 1 for $a \geq a_{NDE}$ as $\epsilon = a_{NDE}$, $\beta \to 0$, and $\alpha \to \infty$.

The resulting probability of crack detection under two NDE systems using the intersection rule is given by

$$F_D(a) = POD(a;1)POD(a;2) \tag{9}$$

For the situation in which only a single inspection is used, $F_D(a)$ is equal to POD(a;1), i.e.,

$$F_D(a) = POD(a;1) \tag{10}$$

III. Formulation

The component is assumed to consist of only one critical location and the failure of such a critical location will imply the failure of the entire component. Furthermore, when a crack is detected in such a critical location, the component will be retired or replaced. A gas turbine engine disk, however, usually contains many critical locations, including bolt holes, rim holes, web holes, etc. The entire disk will be retired and replaced by a new one if a crack is detected in any one of the critical locations. The solution of such a component with multiple critical locations will be described later. For simplicity of derivation and presentation, service inspection maintenance is assumed to be periodic, i.e., the inspection interval is identical, as shown in Fig. 1.

Nonreplacement Policy (Nonrenewal Process)

We shall consider first the nonreplacement policy, in which components are not replaced when they fail in service or when they are retired during service inspection maintenance. Thus, the fleet size decreases monotonically as a function of service time. This is referred to as a nonrenewal process. Solutions under the nonreplacement policy will be derived in this subsection, from which solutions under a replacement policy will be derived later.

Consequently, the probability density function of the crack size described later *is not normalized*, in the sense that the integration does not result in a unity, because the crack population reduces monotonically.

In the First Service Interval $(0,\tau)$

Suppose a crack is initiated at time (cycles) t in $(0,\tau)$, i.e., $a(t) = a_0$. Then the crack size at the end of the jth service interval $j\tau$, denoted by $a(j\tau)$, is obtained by integrating Eq. (3) from t to $j\tau$ as follows:

$$a(j\tau) = a_0/[1 - a_0^c cQ(j\tau - t)Z]^{1/c} \tag{11}$$

in which

$$c = b - 1 \tag{12}$$

Obviously, the crack size $a(\tau)$ at the end of the first service interval τ, i.e., $j = 1$ in Eq. (11), is a random variable since Z is a random variable. The conditional probability density of $a(\tau)$, denoted by $f_{a(\tau)}(x|t)$ given that a crack is initiated at t in $(0,\tau)$, can be obtained from the probability density of Z given by Eq. (6) through the transformation of Eq. (11) for $j = 1$, with the results

$$f_{a(\tau)}(x|t) = f_Z[W(x;\tau - t)]I(x;\tau - t) \quad \text{for } x \geq a_0 \tag{13}$$

where

$$W(x;\tau - t) = [cQ(\tau - t)]^{-1}(a_0^{-c} - x^{-c})$$
$$I(x;\tau - t) = [Q(\tau - t)]^{-1}(x)^{-c-1} \tag{14}$$

The probability density function $f_{a(\tau)}(a;2)$ of the crack size $a(\tau)$ at τ for $x \geq a_0$ can be obtained from the conditional probability density $f_{a(\tau)}(x|t)$ using the theorem of total probability

$$f_{a(\tau)}(x;2) = \int_0^\tau f_Z[W(x;\tau - t)] \times I(x;\tau - t)f_T(t)\mathrm{d}t \quad \text{for } x \geq a_0 \tag{15}$$

where $f_T(t)$ is the probability density function of the time to crack initiation given by Eq. (1).

The probability of failure in the first service interval $(0,\tau)$, denoted by $\bar{p}(1)$, is equal to the probability that the crack size $a(\tau)$ at τ is greater than the critical crack size a_c,

$$\bar{p}(1) = p[a(\tau) \geq a_c] = \int_{a_c}^\infty f_{a(\tau)}(x;2)\mathrm{d}x \tag{16}$$

where $f_{a(\tau)}(x;2)$ is given by Eq. (15).

In the Second Service Interval $(\tau, 2\tau)$

The probability density of the crack size at the end of the second service interval 2τ is contributed by two parts: the first comes from those cracks initiated in the first service interval $(0,\tau)$ but not detected at τ, and the second from those cracks initiated in the second service interval $(\tau,2\tau)$. Each contribution is derived in the following.

Let $a_I(\tau)$ be the crack size at τ, given that the crack is initiated at t in $(0,\tau)$, , i.e., $a(t) = a_0$, and reaches a size x at 2τ.

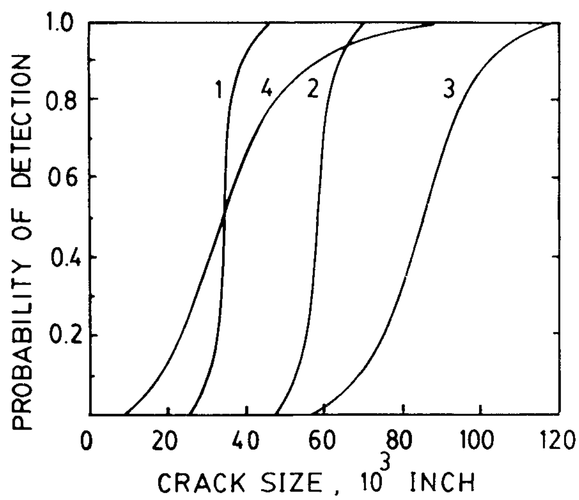

Fig. 2 Various POD curves for NDE system.

Then, it can be shown from Eq. (11) that[18]

$$a_1(\tau) = B(x;\tau - t, 2\tau - t) = a_0 \Big/ \left[1 - \frac{\tau - t}{2\tau - t}\left(1 - \frac{a_0^c}{x^c}\right)\right]^{1/c} \tag{17}$$

Hence, the conditional probability density of the crack size at 2τ, contributed by those cracks initiated at t in $(0,\tau)$, can be obtained from the transform of Eq. (11) for $j=2$, with the results

$$\begin{aligned} f^{(1)}_{a(2\tau)}(x\,|\,t) &= F^*_D[B(x;\tau - t, 2\tau - t)] f_Z[W(x;2\tau - t)] \\ &\quad \times I(x;2\tau - t) \qquad x \geq a_0 \\ &= 0 \qquad B(x;\tau - t, 2\tau - t) \geq a_c \end{aligned} \tag{18}$$

where $W(x;2\tau - t)$ and $I(x;2\tau - t)$ are given by Eq. (14) with τ being replaced by 2τ and $F^*_D(x)$ is the probability of missing the crack size x during the service inspection, i.e.,

$$F^*_D(x) = 1 - F_D(x) \tag{19}$$

where $F_D(x)$ is given by Eq. (9). The second equation of Eq. (18) indicates that the component will have failed in $(0,\tau)$ if $a_1(\tau) \geq a_c$.

The unconditional probability density function is obtained as

$$\begin{aligned} f^{(1)}_{a(2\tau)}(x;2) &= \int_0^\tau F^*_D[B(x;\tau - t, 2\tau - t)] f_Z[W(x;2\tau - t)] \\ &\quad \times I(x;2\tau - t) f_T(t)\,\mathrm{d}t; \qquad x \geq a_0 \end{aligned} \tag{20}$$

The probability density function contributed by those cracks initiated in the second service interval $(\tau, 2\tau)$ can be obtained in a similar fashion as Eq. (15)[18]

$$\begin{aligned} f^{(2)}_{a(2\tau)}(x;2) &= \int_0^\tau f_Z[W(x;\tau - t)] \\ &\quad \times I(x;\tau - t) f_T(\tau + t)\,\mathrm{d}t; \quad x \geq a_0 \end{aligned} \tag{21}$$

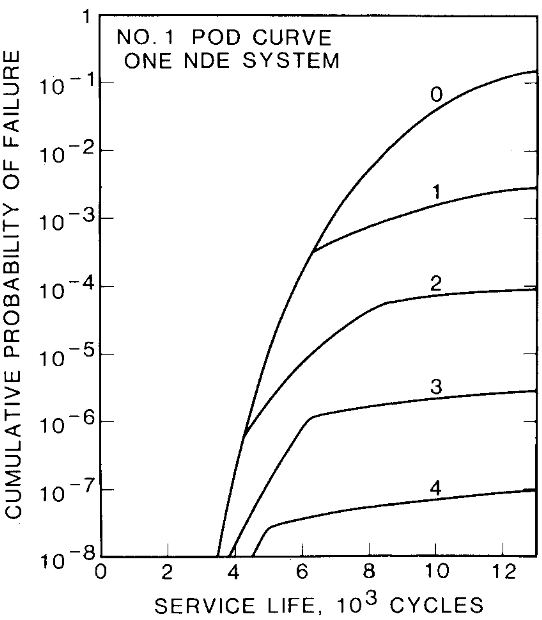

Fig. 3 Cumulative probability of failure as function of service life using POD curve 1.

Consequently, the probability density function for the crack size greater than a_0 at the end of the second service interval 2τ is the summation of Eqs. (20) and (21),

$$f_{a(2\tau)}(x;2) = f^{(1)}_{a(2\tau)}(x;2) + ff^{(2)}_{a(2\tau)}(x;2) \tag{22}$$

The probability of failure for a component in the second service interval, denoted by $\bar{p}(2)$, is obtained as

$$\bar{p}(2) = \int_{a_c}^{\infty} f_{a(2\tau)}(x;2)\,\mathrm{d}x \tag{23}$$

In the nth Service Interval $[(n-1)\tau, n\tau]$

In a similar fashion, the probability density function for the crack size larger than a_0 at the end of the nth service interval $n\tau$ prior to inspection can be derived as follows:

$$f_{a(n\tau)}(x;2) = \sum_{i=1}^{n} f^{(i)}_{a(n\tau)}(n\tau)(x;2); \quad x \geq a_0 \tag{24}$$

where $f^{(i)}_{a(n\tau)}(x;2)$ is the contribution from cracks initiated in the ith service interval given by[18]

$$\begin{aligned} f^{(i)}_{a(n\tau)}(x;2) &= \int_0^\tau \left\{ \prod_{m=1}^{n-i} \mathrm{F}^*_D[B(x;m\tau - t, (n-i+1)\tau - t)] \right\} \\ &\quad \times f_Z[W(x;(n-i+1)\tau - t)] \\ &\quad \times I[x;(n-i+1)\tau - t] f_T[(i-1)\tau + t]\,\mathrm{d}t \end{aligned} \tag{25}$$

where $F^*_D[B] = 0$ for $B > a_c$ and

$$\prod_{m=1}^{n-i} \mathrm{F}^*_D[B(x;m\tau - t, (n-i+1)\tau - t)] = 1 \quad \text{for } n = i \tag{26}$$

$$\begin{aligned} &B[x;m\tau - t, (n-i+1)\tau - t] \\ &\quad = a_0 \Big/ \left[1 - \frac{m\tau - t}{(n-i+1)\tau - t}\left(1 - \frac{a_0^c}{x^c}\right)\right]^{1/c} \end{aligned} \tag{27}$$

$$W[x;(n-i+1)\tau - t] = \{cQ[(n-i+1)\tau - t]\}^{-1}(a_0^{-c} - x^{-c}) \tag{28}$$

$$I[x;(n-i+1)\tau - t] = \{Q[(n-i+1)\tau - t]\}^{-1}(x)^{-c-1} \tag{29}$$

The probability of failure in the nth service interval is given by

$$\bar{p}(n) = \int_{a_c}^{\infty} f_{a(n\tau)}(x;2)\,\mathrm{d}x \tag{30}$$

Probabilities of Retirement and *K* Account

In addition to the probability of failure derived above, the probability of retirement (or crack detection) $\bar{G}(n)$ during the nth inspection maintenance and the probability $\bar{K}(n)$ that a component will be inspected by the second NDE system (i.e., inspected twice), referred to as the probability of K account, are important in the cost/risk analysis of engine components. The former is related to the cost of replacement, whereas the later is related to the cost of employing the second NDE system. In order to estimate these probabilities, the probability density function of the crack size in the region smaller than a_0 should be determined. This is because the first NDE system may detect a crack size smaller than a_0, although such a contribution is usually very small. The probability density function of the crack size $a(t)$ in the crack size region smaller than a_0 has been derived in Ref. 18, with the results

$$f_{a(t)}(x;1) = \frac{\log e}{\sqrt{2\pi} v x \tilde{\sigma}_T} \times \exp\left\{-\frac{1}{2}\left(\frac{-(1/v)\log(x/a_0 t^v) - \tilde{\mu}_T}{\tilde{\sigma}_T}\right)^2\right\}; \quad x < a_0 \quad (31)$$

in which $v = \tilde{\sigma}_I/\tilde{\sigma}_T$ is the ratio of the standard deviations of log $a(0)$ and log T, respectively, where $a(0)$ is the initial crack size and T the time to crack initiation, the distribution of which is given by Eq. (1). The initial crack size $a(0)$ is also assumed to follow the log normal distribution. It is mentioned that $\tilde{\sigma}_I$ and $\tilde{\sigma}_T$ are the measures of the statistical dispersions of $a(0)$ and T, respectively, and that they have been observed to be very close to each other.[18]

Thus, the probability density of the crack size prior to each service inspection maintenance $f_{a(n\tau)}(x)$ for $n = 1,2,...$ is expressed as $f_{a(n\tau)}(x) = f_{a(n\tau)}(x;2)$ for $x \geq a_0$ and $f_{a(n\tau)}(x) = f_{a(n\tau)}(x;1)$ for $x < a_0$ given by Eqs. (24) and (31), respectively. During the nth inspection maintenance, the probability that a component will be retired (i.e., a crack will be detected), denoted by $\bar{G}(n)$, and the probability of K account, denoted by $\bar{K}(n)$, are given by

$$\bar{G}(n) = \int_0^{a_c} f_{a(n\tau)}(x) F_D(x) \mathrm{d}x$$

$$\bar{K}(n) = \int_0^{a_c} f_{a(n\tau)}(x) \mathrm{POD}(x;1) \mathrm{d}x \quad (32)$$

It is emphasized that the contribution to $\bar{G}(n)$ and $\bar{K}(n)$ from $f_{a(n\tau)}(x;1)$ is very small and hence the approximation given by Eq. (31) is not at all critical.

Replacement Policy (Renewal Process)

Under the replacement policy, when a component fails in service, it is replaced by a new component at the end of the service interval. Likewise, a component is also replaced by a new one when it is retired during each service inspection maintenance. Thus, a constant fleet size is maintained. The new component for replacement is referred to as the renewal population and the replacement policy is also referred to as the renewal process. Under the replacement policy, let $R(j)$ be the probability that a component will be replaced at the end of the jth service interval, $p(j)$ the probability of failure in the jth service interval under the replacement policy, and $K(j)$ the probability of K account. These quantities can be derived from the results obtained previously under the nonreplacement policy.

The probability of replacement $R(1)$ at the end of the first service interval τ consists of the probability of failure and the probability of retirement

$$R(1) = \bar{G}(1) + \bar{p}(1) \quad (33)$$

At the end of the second service interval 2τ, the probability of replacement consists of the probability of failure in $(\tau, 2\tau)$ and the probability of retirement at 2τ for both the original population and the renewal population introduced at τ with probability $R(1)$,

$$R(2) = [\bar{G}(2) + \bar{p}(2)] + R(1)[\bar{G}(1) + \bar{p}(1)] \quad (34)$$

where $[\bar{G}(1) + \bar{p}(1)]$ is the probability of replacement at 2τ for the renewal population introduced at τ [with probability $R(1)$].

In a similar manner, the probability of replacement at the end of the jth service interval $j\tau$ can be obtained as

$$R(j) = [\bar{G}(j) + \bar{p}(j)] + \sum_{k=1}^{j-1} R(k)[\bar{G}(j-k) + \bar{p}(j-k)] \quad \text{for } j = 2,3,... \quad (35)$$

Equation (35) is a recurrent solution that can be computed starting from $R(1)$ given by Eq. (33).

In a similar manner, the probability of K account and the probability of failure under the replacement policy can be obtained as

$$K(1) = \bar{K}(1); \quad K(j) = \bar{K}(j) + \sum_{k=1}^{j-1} R(k)\bar{K}(j-k) \quad \text{for } j = 2,3,... \quad (36)$$

$$p(1) = \bar{p}(1); \quad p(j) = \bar{p}(j) + \sum_{k=1}^{j-1} R(k)\bar{p}(j-k) \quad \text{for } j = 2,3,... \quad (37)$$

Since the probability of failure of fatigue critical components is usually designed to be very small, it is reasonable to assume that the event of failure in each service interval is statistically independent. The cumulative probability of failure in the service interval $(o, j\tau)$, denoted by $P(j\tau)$, is obtained as

$$P(j\tau) = 1 - \prod_{k=1}^{j} [1 - p(k)] \quad j = 1,2,... \quad (38)$$

IV. Demonstrative Example

The analysis methodology developed herein is applied to the third-stage turbine disk of a TF-33 gas turbine engine for demonstrative purpose. The material of the engine disk is Incoloy 901 and the 10 bolt holes are the critical locations where cracks may develop. Currently, the design life for such a disk is 2500 cycles, i.e., the disk is discarded after 2500 cycles of service life. Under service inspection maintenance, the design life is extended to 12,500 cycles.

Available test results indicate that the initial fatigue quality can be represented by the distribution of time to crack initiation (TTCI) with the reference crack size $a_0 = 0.03$ in. The median life and the coefficient of variation of TTCI are, respectively, $\tilde{T} = 4000$ cycles and $V_T = 75\%$. The log normal parameters appearing in Eq. (1) for TTCI distribution are computed at $\tilde{\mu}_T = \log \tilde{T} = \log 4000 = 3.6$ and $\tilde{\sigma}_T = \sqrt{\ell n(1 + V_T^2)}/\ell n 10 = 0.29$. Crack propagation data at two bolt hole locations were obtained from spin pit tests under a nominal loading spectrum.[4] The average crack propagation life from the reference crack size $a_0 = 0.76$ mm (0.03 in.) to the critical crack size $a_c = 8.51$ mm (0.355 in.) is approximately 11,500 cycles.[4] The crack growth data under the nominal loading spectrum ($S = 1.0$) are best fitted by the crack growth rate equation (2) with the result, $Q = 4.22 \times 10^{-4}$ and $b = 1.2914$. On the basis of other experimental data, the statistical dispersion of the crack growth rate is $V_Z = 30\%$, as defined by that of the random variable Z in Eq. (4). Therefore, the log normal parameter for Z is $\tilde{\sigma}_z = 0.1276$, see Eq. (6). Furthermore, the parameter v appearing in Eq. (31) is approximately 1.0469.

The probability of crack detection for the first NDE system is defined by Eq. (7). A POD curve with $\alpha^* = 55.28$ and $\beta^* = 16.4$ shown in Fig. 2 as curve 1 is considered. Since such a POD curve is narrow banded, the second NDE system is not employed.

Without inspection maintenance, the cumulative probability of failure $P(j\tau)$ for one bolt hole is computed as a function of service life; the result is shown in Fig. 3 by a solid curve designated as curve 0. It is observed that the cumulative probability of failure increases as the service life increases.

Under the scheduled inspection maintenance, the cumulative probabilities of failure $P(j\tau)$ for a bolt hole are computed and presented in Fig. 3 by solid curves designated 1-4. The number shown on each curve (or curve number) denotes the number of inspection maintenances performed in the service life of 12,500 cycles. For example, curve 3 in Fig. 3 represents the cumulative probability of failure for a bolt hole in which three service inspection maintenances are performed in 12,500 cycles, i.e., the inspection interval $\tau = 3125$ cycles.

Figure 3 clearly demonstrates that the probability of failure in service is reduced significantly by the application of scheduled inspection maintenance. Furthermore, the probability of failure decreases as the number of inspection maintenances in 12,500 cycles increases.

The probability of replacement $R(j)$ during the jth inspection maintenance for $j = 1,2,...,$ can be interpreted as the average percentage of replacement. The results have been computed and presented in Table 1. Also presented in Table 1 is the total average percentage of replacements, i.e., $R =$ sum of $R(j)$ from $j = 1$ to $n-1$. It is observed from Table 1 that the total average percentage of replacements increases as the number of inspection maintenances increases.

The purpose of employing the second NDE system is to return good components rejected by the first NDE system into service, thus reducing the percentage (and cost) of replacement. Such an advantage, however, is compensated by the cost of the K account, since the second inspection is usually expensive and time consuming. As a result, the advantage of the second NDE system will be more significant when the first NDE system involves more detection uncertainty, i.e., the POD curve is wide banded. Since the POD curve of the first NDE system is narrow banded (less detection uncertainty) (see curve 1 of Fig. 2), the application of the second NDE system is not advantageous at all.

Let the POD curve of the first NDE system be the one shown by curve 2 in Fig. 2 in which $\alpha^* = 66.6$ and $\beta^* = 23.4$. Although such an NDE system is not as capable as the previous one, the POD curve is narrow banded, indicating less statistical uncertainty in crack detection. Consequently, the second NDE system is not employed. The cumulative probabilities of failure are displayed by solid curves in Fig. 4 for different inspection intervals. The average percentage of replacements during each inspection maintenance and the total average percentage of replacements in the service life of 12,500 cycles are presented in Ref. 18.

It can be observed from Figs. 3 and 4 that, with the same inspection interval τ, the probability of failure using POD curve 2 is much higher than that using POD curve 1. This has been expected because POD curve 1 is capable of detecting smaller cracks. However, the total average percentage of replacements is smaller with the application of POD curve 2 (see Ref. 18).

Consider POD curve 3 as shown in Fig. 2 for the first NDE system where $\alpha^* = 28.94$ and $\beta^* = 11.73$. It is obvious that the capability of such an NDE system is not as good as other POD curves. By use of one NDE system, the cumulative prob-

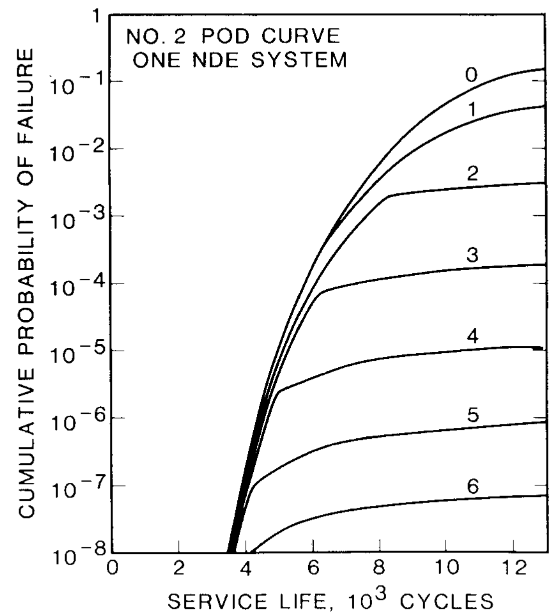

Fig. 4 Cumulative probability of failure as function of service life using POD curve 2.

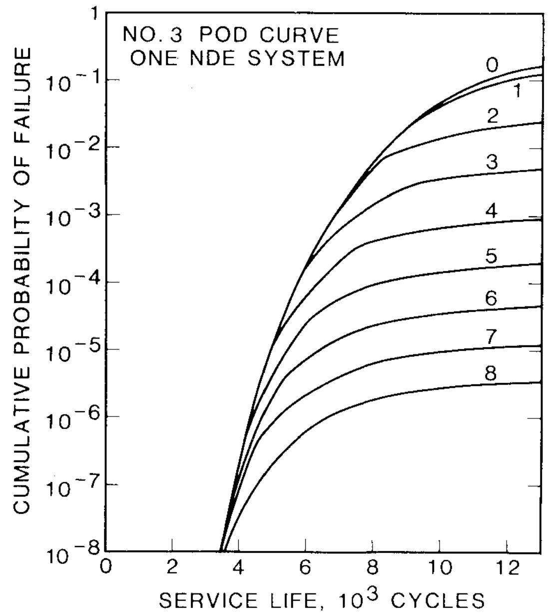

Fig. 5 Cumulative probability of failure as function of service life using POD curve 3.

Table 1 Average percentage of replacement using POD curve 1

No. of inspec.	Inspec. interval, cycles	Average percentage of replacements, jth inspection maintenance 1	2	3	4	5	6	7	8	Total
1	6250	65.43								65.43
2	4166	36.64	58.59							95.23
3	3125	19.07	50.09	39.72						108.88
4	2500	9.91	40.86	34.87	34.95					120.59
5	2083	5.29	31.62	32.38	29.55	30.28				129.12
6	1785	2.93	23.64	29.93	26.39	26.22	26.69			135.80
7	1562	1.69	17.37	26.93	24.51	23.23	23.64	23.84		141.21
8	1388	1.00	12.68	23.58	23.07	21.19	21.15	21.47	21.53	147.47

Table 2 Average percentage of replacement and K-account using POD curve 4

No. of inspec.	Inspec. interval, cycles	Average percentage of replacements, *i*th inspection maintenance: 1	2	3	4	5	6	Two-NDE system total	One-NDE system total
1	6250	19.82						19.82	62.24
2	4166	1.62	47.83					49.45	100.65
3	3125	0.09	19.73	42.18				62.00	127.31
4	2500	0.0	6.12	31.72	30.82			68.66	149.92
5	2083	0.0	1.63	18.19	29.78	23.05		72.65	170.29
6	1785	0.0	0.40	8.84	23.41	24.59	18.26	75.50	189.43

No. of inspec.	Inspec. interval, cycles	Average percentage of K account, *i*th inspection maintenance: 1	2	3	4	5	6	Total
1	6250	62.24						62.24
2	4166	39.19	76.88					116.07
3	3125	25.85	62.11	69.12				157.08
4	2500	17.80	49.12	67.58	57.17			191.67
5	2083	12.66	39.18	60.77	61.36	50.32		224.29
6	1785	9.25	31.57	52.73	61.66	54.21	46.51	255.93

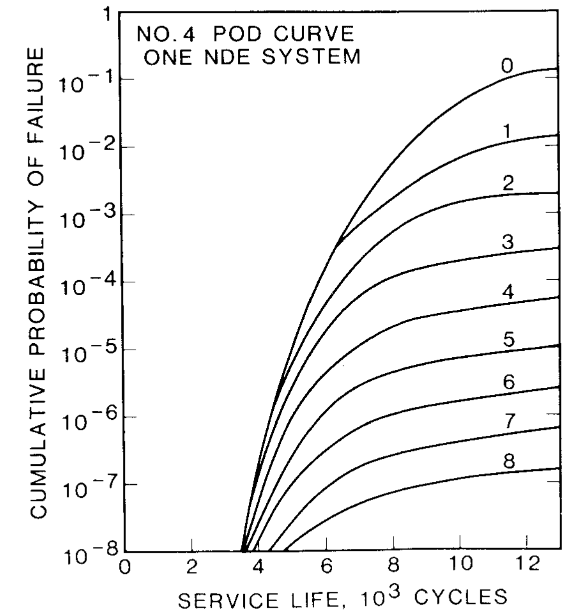

Fig. 6 Cumulative probability of failure as function of service life using POD curve 4.

abilities of failure are shown in Fig. 5. Although the capability of such an NDE system is not outstanding, it is still possible to substantially reduce the failure probability using a shorter inspection interval τ. The percentage of replacement during each inspection maintenance was presented in Ref. 18. As expected, the total average percentage of replacement is much smaller than those associated with POD curves 1 and 2.

Finally, POD curve 4 shown in Fig. 2 is considered for the first NDE system, where $\alpha^* = 13.44$ and $\beta^* = 3.95$. With a single NDE system, the cumulative probabilities of failure are displayed in Fig. 6. The average total percentage of replacement in 12,500 cycles is shown in the right-hand column of Table 2. (also see Ref. 18.)

As observed from Fig. 2, POD curve 4 is wide banded, involving considerable statistical uncertainty in crack detection. Hence, the second NDE system is employed in which the POD curve is considered to be a unit step function at the crack size $a = \epsilon = 0.06$ in., which corresponds to a 90% detection probability for the first NDE system. The cumulative probabilities of failure have been computed but not displayed because of space limitations. It is mentioned that the failure probability using two NDE systems is slightly higher than that shown in Fig. 6, in which only one NDE system is employed. The average percentage of replacements as well as the average percentage of the K account are given in Table 2. A comparison between the total average percentages of replacement shown in Table 2 indicates that the second NDE system is capable of significantly reducing the total percentage of replacement.

V. Conclusion and Discussion

A probabilistic method based on fracture mechanics has been developed for the reliability analysis of gas turbine engine components under scheduled inspection maintenance in service. Using the third-stage turbine disk of a TF-33 jet engine as an example, it is demonstrated that service inspection maintenances can significantly reduce the probability of failure. The improvement of fatigue reliability is shown to be sensitive to the crack detection capability of the NDE system employed. With appropriate NDE systems, it is concluded that the design life of gas turbine engine disks can be extended considerably, while maintaining a high level of reliability through the application of scheduled inspection maintenances. A tremendous life-cycle-cost saving and a substantial reduction in disk replacements resulting from scheduled inspection maintenances has been reported.[19]

In addition to jet engine components, the present methodology can be applied to much wider practical problems, such as large generators in hydraulic or nuclear powerplants, various types of machinery under fatigue environments, helicopters, offshore structures, etc., to mention just a few.

In the present formulation, various quantities involving statistical uncertainties have been taken into account. These include the statistical variabilities of materials, time to crack initiation, crack propagation, service loads, stress intensity factor, stress concentration factor, crack geometry, crack modeling, idealization of equivalent load cycles, NDE capability, etc. However, several statistical quantities are not accounted for as explained in the following paragraphs.

Experimental test results indicate that the fracture toughness K_c involves statistical scatter. Hence, the critical crack size a_c, theoretically, should be considered as a random variable. However, as the crack size approaches the critical crack size a_c, the crack growth rate is very high and the crack growth damage accumulates very rapidly. Thus, the variability of the critical crack size has a negligible effect on the fatigue life of the component.

The number of load cycles in each service interval τ cannot be determined precisely because of possible miscount. However, this uncertainty can be reduced significantly by a better fleet tracking system. Likewise, it can be accounted for in terms of the crack propagation rates, such as H_4 described previously.

The solutions obtained above hold for a critical location, such as one bolt hole. A turbine disk usually contains many holes and the entire disk will fail if any one hole fails. Hence, the survival (or the reliability) of the entire disk implies the survival of all holes. If the survival of each hole is assumed to be satistically independent for a conservative estimate, then the probability of failure of the entire disk can be computed easily.[11,18] On the other hand, the solutions obtained in this paper may be used in approximation for a disk, if 1) the distribution of the time to crack initiation is the smallest among all holes, 2) the distribution of the crack growth damage accumulation is the largest among all holes, and 3) the POD curve represents that for a disk. While the distribution in assumptions 1 and 2 can be derived using the extreme value theory, further research is needed to establish a POD curve for a disk.

In the present investigation, the initial fatigue quality is represented by the distribution of the time to crack initiation alone. This is applicable to some engine materials, such as titanium, Waspaloy, Astroloy, etc. Recently, the NDE capability has been improved significantly such that a small crack size can be detected with high probability. Furthermore, intensive research efforts have been conducted to predict the fatigue crack growth behavior in the small crack size region. As a result, the reference crack size a_0 at crack initiation can be defined to be very small, thus increasing tremendously the fatigue crack propagation life. This is particularly true for some powder metallurgy superalloys. Under such a circumstance, however, there are initial flaws that are greater than a_0 and that start to propagate immediately after the component is introduced into service, i.e., the TTCI is zero. Consequently, the initial fatigue quality should be represented by two populations: one defined by the distribution of the time to crack initiation and the other by that of the initial flaw size. The solution for such a general case is presented in Ref. 19.

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References

[1]Cowies, W. P., "An Overview of Turbine Engine Structural Design, Development Trends in the Air Force," *Proceedings of AIAA/ASME/ASCE/AHS 24th SDM Conference,* 1983, pp. 493-498.

[2]Cowie, W. P. and Stein, T. A., "Damage Tolerant Design and Test Considerations in the Engine Structural Integrity Program," *Proceedings of AIAA/ASME/ASCE/AHS 21st SDM Conference,* 1980, pp. 723-728.

[3]King, T. T., "USAF Engine Structural Durability and Damage Tolerance," AIAA Professional Study Series, Seattle, June 30-July 1, 1983.

[4]Hill, R. J., Reimann, W. H., and Ogg, J. S., "A Retirement-For-Cause Study of an Engine Turbine Disk," AFWAL-TR-81-2094, Nov. 1981.

[5]Annis, C. G. Jr., Van Wanderham, M. C., Harris, J. A. Jr., and Sims, D. L., "Gas Turbine Engine Disk Retirement-For-Cause; An Application of Fracture Mechanics and NDE," *Journal of Engineering for Power, Transactions of ASME,* Vol. 103, No. 1, Jan. 1981, pp. 198-200.

[6]Besuner, P. M. et al., "Cost/Risk Analysis for Disk Retirement: Vol. I," Failure Analysis Associates, Palo Alto, Calif., Rept. FAA-82-8, 1983.

[7]Besuner, P. M. et al., "Analysis Methods for F100 Engine Component Retirement for Cause," Failure Analysis Associates, Palo Alto, Calif., Rept. FAA-81-12-5, 1982.

[8]Harris, J. A. et al., "Engine Component Retirement for Cause," Pratt & Whitney Aircraft Group, Government Product Division, West Palm Beach, Fla., Contract F33615-80-C-5160, 1980-1985.

[9]Eggwertz, S., "Investigation of Fatigue Life and Residual Strength of Wing Panel for Reliability Purposes," *Probabilistic Aspects of Fatigue,* ASTM-STP 511, ASTM, Philadelphia, 1972, pp. 75-105.

[10]Eggwertz, S. and Lindsjö, G., "Influence of Detected Crack Length at Inspections on Probability of Fatigue Failure of Wing Panel," Aeronautical Research Institute of Sweden, Tech. Note HU-1745, Pt. 2, ICAF No. 834, Stockholm, 1975.

[11]Yang, J. N. and Trapp, W. J., "Reliability Analysis of Aircraft Structures Under Random Loading and Periodic Inspection," *AIAA Journal,* Vol. 12, Dec. 1974, pp. 1623-1630.

[12]Yang, J. N. and Trapp, W. J., "Inspection Frequency Optimization for Aircraft Structures Based on Reliability Analysis," *Journal of Aircraft,* Vol. 12, May 1975, pp. 494-496.

[13]Yang, J. N., "Reliability Analysis of Structures Under Periodic Proof Test in Service," *AIAA Journal,* Vol. 14, Sept. 1976, pp. 1225-1234.

[14]Yang, J. N., "Statistical Estimation of Service Cracks and Maintenance Cost for Aircraft Structures," *Journal of Aircraft,* Vol. 12, Dec. 1975, pp. 929-937.

[15]Yang, J. N., "Optimal Periodic Proof Test Based on Cost-Effective and Reliability Criteria," *AIAA Journal,* Vol. 15, March 1977, pp. 402-409.

[16]Yang, J. N., "Statistical Approach to Fatigue and Fracture Including Maintenance Procedures," *Fracture Mechanics,* University Press of Virginia, Charlottesville, Sept. 1978, pp. 559-577.

[17]Yang, J. N., "Statistical Estimation of Economic Life for Aircraft Structures," *Proceedings of AIAA/ASME/ASCE/AHS 20th SDM Conference,* 1979, pp. 240-248 (also *Journal of Aircraft,* Vol. 17, July 1980, pp. 528-535).

[18]Yang, J. N. and Chen, S., "Fatigue Reliability of Gas Turbine Engine Components Under Scheduled Inspection Maintenance," *Proceedings of AIAA/ASME/ASCE/AHS 25th SDM Conference,* 1984, pp. 410-420.

[19]Yang, J. N. and Chen, S., "An Exploratory Study of Retirement-For-Cause for Gas Turbine Engine Components," AIAA Paper 84-1220, June 1984.

[20]Yang, J. N., Salivar, G. C., and Annis, C. G., "Statistical Modeling of Fatigue Crack Growth in a Nickel-Based Super-Alloy," *Journal of Engineering Fracture Mechanics,* Vol. 18, No. 2, June 1983, pp. 257-270.

[21]Schwartz, B., Annis, C. G., Salivar, G. C., and Yang, J. N., "Statistics of Crack Growth in Engine Materials—Vol. II: Spectrum Loading and Advanced Technique," AFWAL-TR-82-4040, WPAFB, Feb. 1983.

[22]Yang, J. N. and Donath, R. C., "Statistical Fatigue Crack Propagation in Fastener Hole Under Spectrum Loading," *Journal of Aircraft,* Vol. 20, Dec. 1983, pp. 1028-1032.

[23]Packman, P. F. et al., "Reliability of Flaw Detection by Nondestructive Inspection," *ASM Metal Handbook,* Vol. 11, 8th Ed., American Society of Metals, Metals Park, Ohio, 1976, pp. 214-224.

[24]Yee, B. G. W. et al., "Assessment of NDE Reliability Data," NASA CR-134991, 1976.

[25]Berens, A. P. and Hovey, P. W., "Evaluation of NDE Reliability Characterization," AFWAL-TR-81-4160, WPAFB, Nov. 1981.

[26]Yang, J. N. and Donath, R. C., "Improving NDE Capability Through Multiple Inspections With Application to Gas Turbine Engine," AFWAL-TR-82-4111, WPAFB, Aug. 1982.

[27]Yang, J. N. and Donath, R. C., "Improving NDE Reliability Through Multiple Inspections," *Review of Progress in Quantitative Nondestructive Evaluation,* Vol. 2A, Plenum Press, New York, 1983, pp. 69-78.